Digital Image Processing

Image Enhancement:
Filtering in the Frequency Domain

Contents

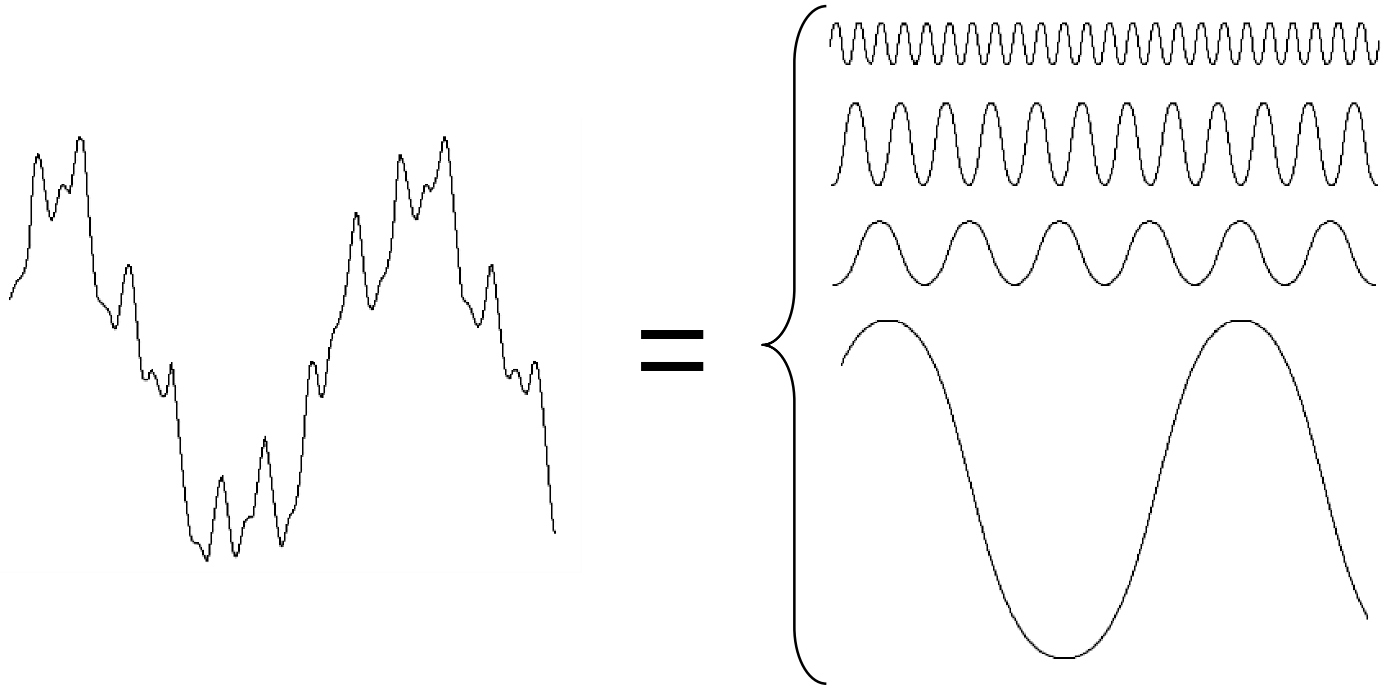
In this lecture we will look at image enhancement in the frequency domain

- Jean Baptiste Joseph Fourier
- The Fourier series & the Fourier transform
- Image Processing in the frequency domain
  - Image smoothing
  - Image sharpening
- Fast Fourier Transform

Jean Baptiste Joseph Fourier

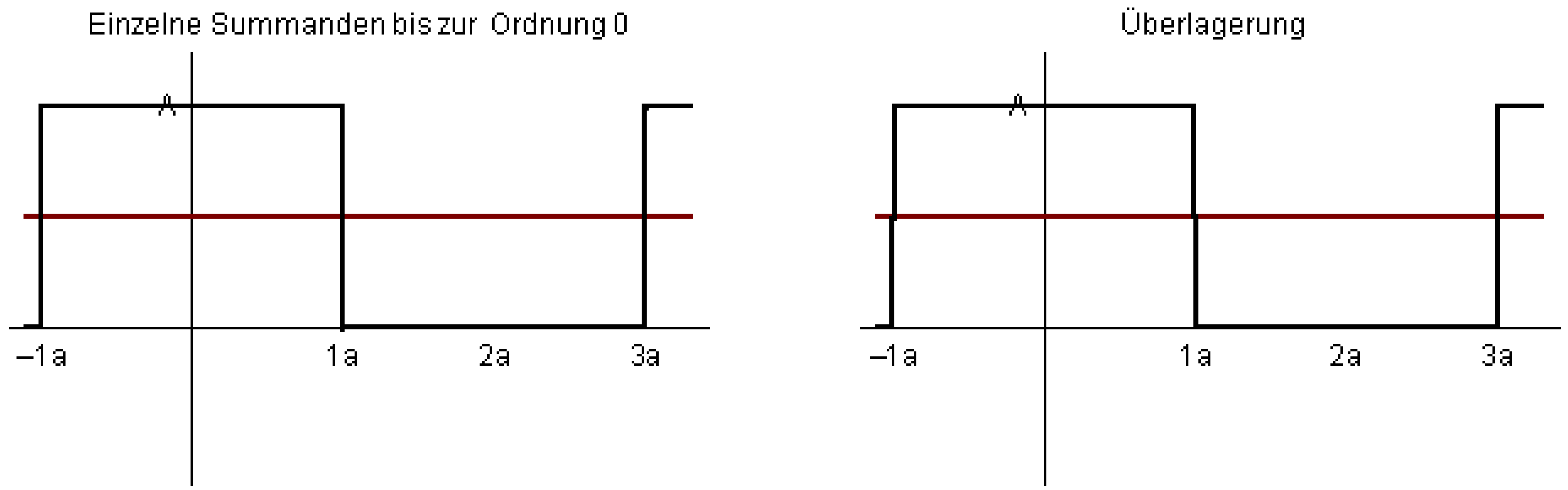
- Fourier was born in Auxerre, France in 1768
  - Most famous for his work *"La Théorie Analitique de la Chaleur"* published in 1822
  - Translated into English in 1878: "*The Analytic Theory of Heat*"
- Nobody paid much attention when the work was first published
- One of the most important mathematical theories in modern engineering

The Big Idea



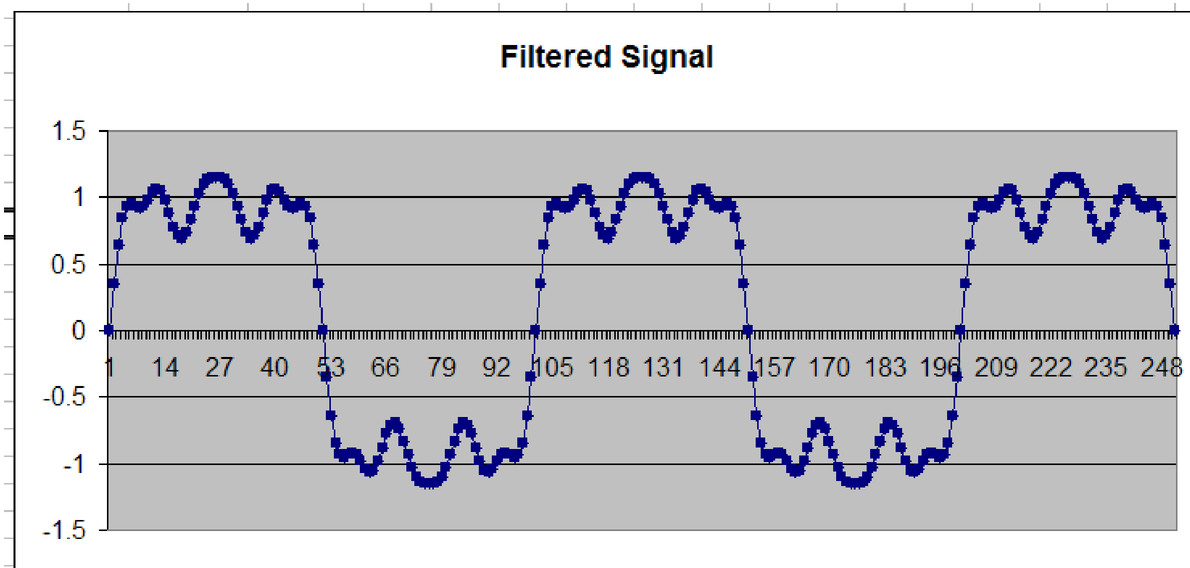
Any function that periodically repeats itself can be expressed as a sum of sines and cosines of different frequencies each multiplied by a different coefficient – a *Fourier series*

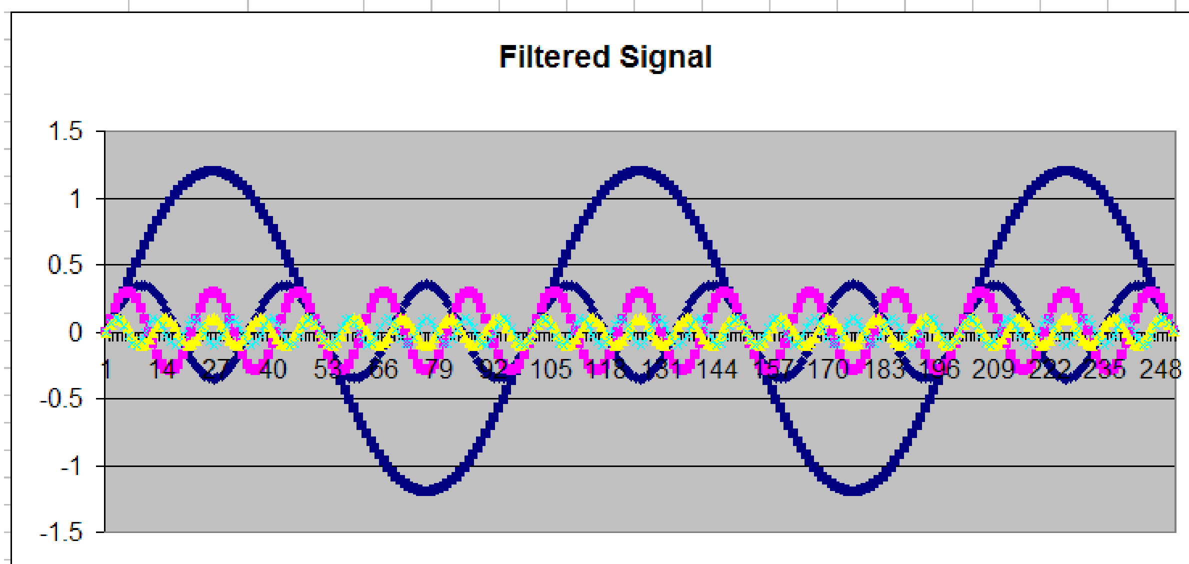
The Big Idea (cont…)



Notice how we get closer and closer to the original function as we add more and more frequencies

The Big Idea (cont…)





Frequency domain signal processing example in Excel

The Discrete Fourier Transform (DFT)

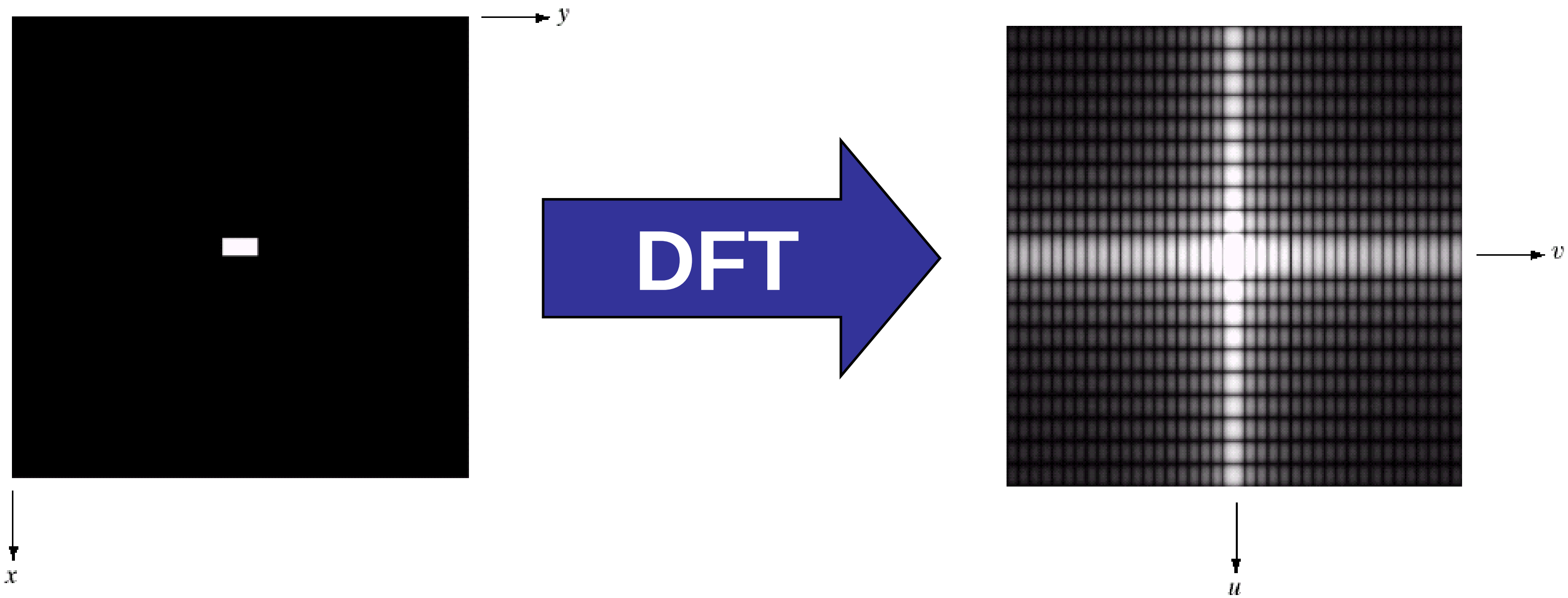
The *Discrete Fourier Transform* of $f(x, y)$, for $x = 0, 1, 2 \ldots M-1$ and $y = 0,1,2 \ldots N-1$, denoted by $F(u, v)$, is given by the equation:

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi(ux/M + vy/N)}$$

for $u = 0, 1, 2 \ldots M-1$ and $v = 0, 1, 2 \ldots N-1$.

DFT & Images

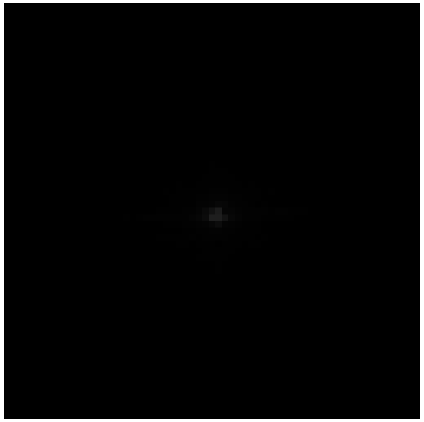
The DFT of a two dimensional image can be visualised by showing the spectrum of the images component frequencies



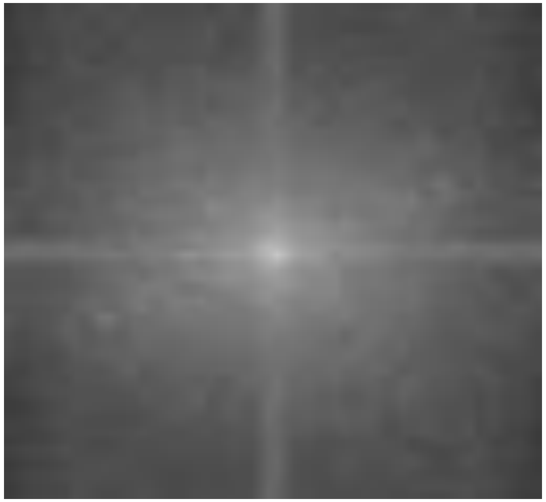
Fourier Transform

- We start off by applying the Fourier Transform of

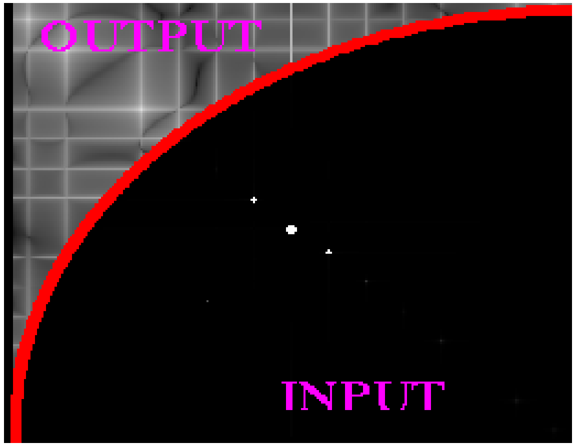
- The magnitude calculated from the complex result is shown in



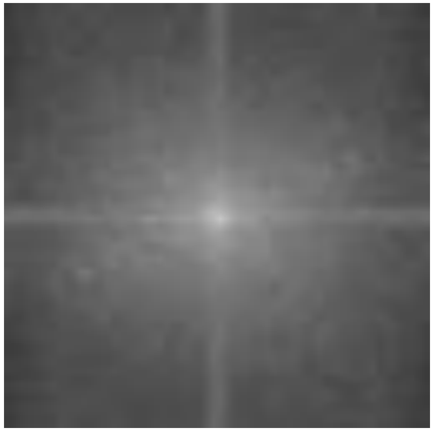
The DC-value is by far the largest component of the image.
However, the dynamic range of the Fourier coefficients (*i.e.* the intensity values in the Fourier image) is too large to be displayed on the screen, therefore all other values appear as black. If we apply a logarithmic transformation to the image we obtain



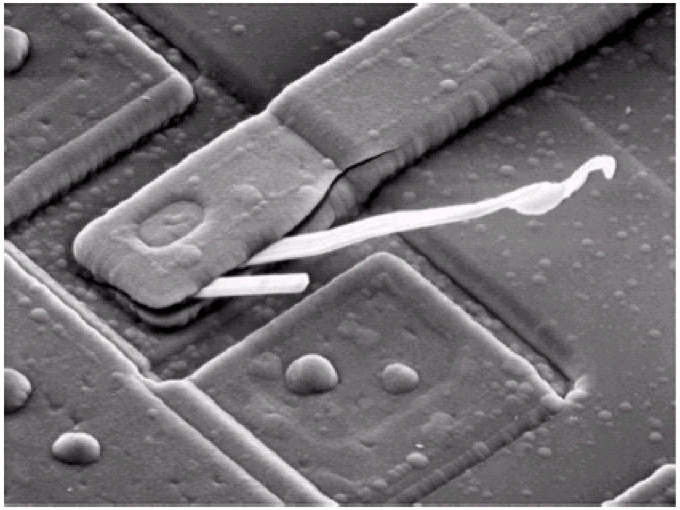
The dynamic range of an image can be compressed by replacing each pixel value with its logarithm. This has the effect that low intensity pixel values are enhanced. Applying a pixel logarithm operator to an image can be useful in applications where the dynamic range may too large to be displayed on a screen (or to be recorded on a film in the first place).



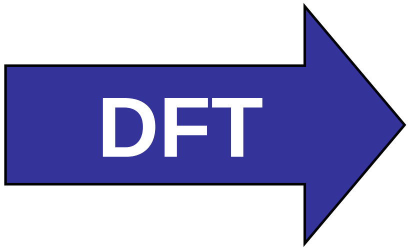
- The result shows that the image contains components of all frequencies,
- Their magnitude gets smaller for higher frequencies. Hence, low frequencies contain more image information than the higher ones.
- The transform image also tells us that there are two dominating directions in the Fourier image, one passing vertically and one horizontally through the center.
- These originate from the regular patterns in the background of the original image.

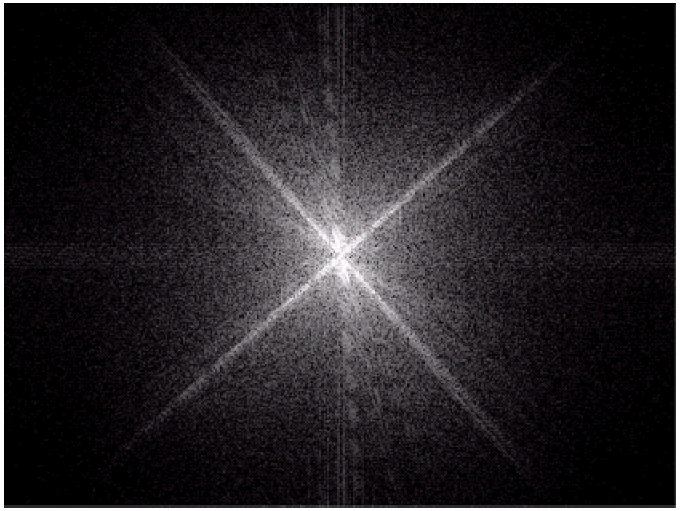


DFT & Images (cont…)



Scanning electron microscope image of an integrated circuit magnified ~2500 times





Fourier spectrum of the image

Features from an image can even sometimes be seen in the Fourier spectrum of the image

The Inverse DFT

It is really important to note that the Fourier transform is completely **reversible**

The inverse DFT is given by:

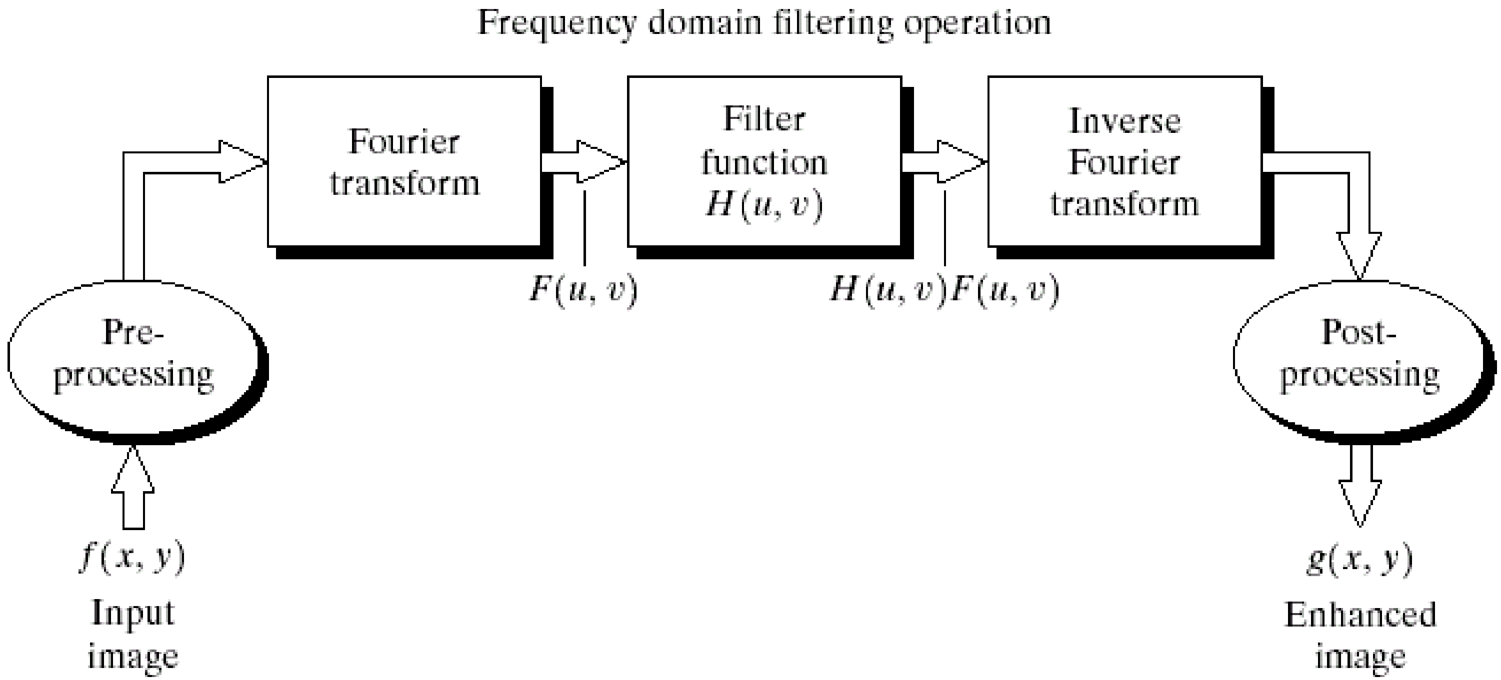
$$f(x, y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) e^{j2\pi(ux/M + vy/N)}$$

for x = 0, 1, 2…M-1 and y = 0, 1, 2…N-1

The DFT and Image Processing

To filter an image in the frequency domain:

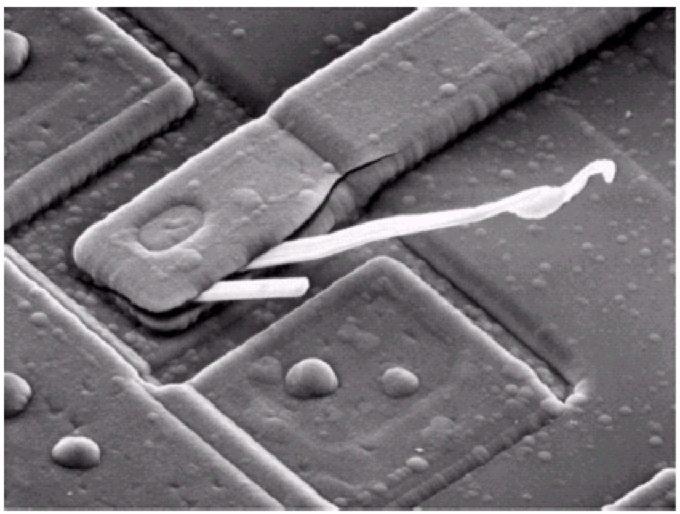
1. Compute $F(u,v)$ the DFT of the image
2. Multiply $F(u,v)$ by a filter function $H(u,v)$
3. Compute the inverse DFT of the result

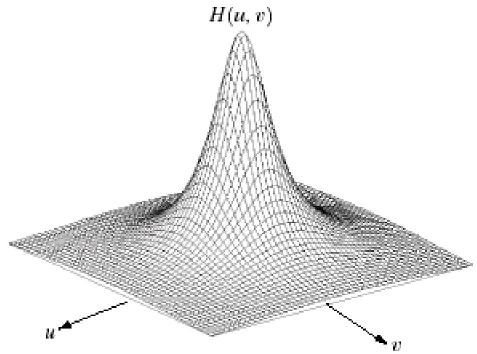


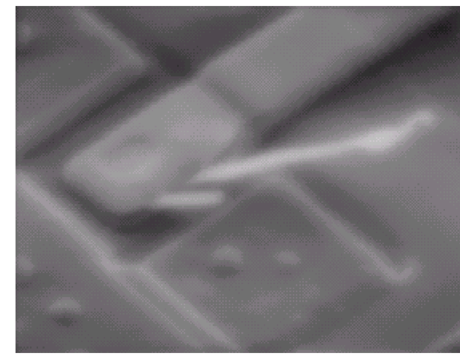
Some Basic Frequency Domain Filters

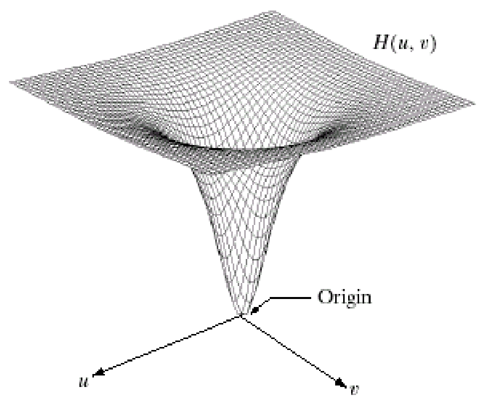
Images taken from Gonzalez & Woods, Digital Image Processing (2002)

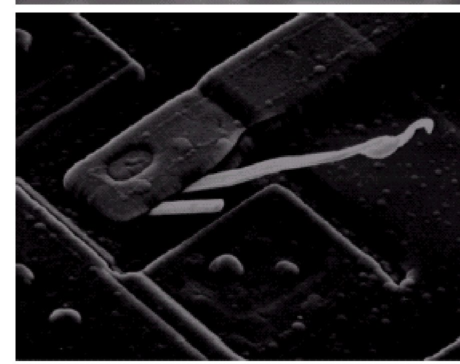
Low Pass Filter











High Pass Filter

Smoothing Frequency Domain Filters

Smoothing is achieved in the frequency domain by dropping out the high frequency components

The basic model for filtering is:

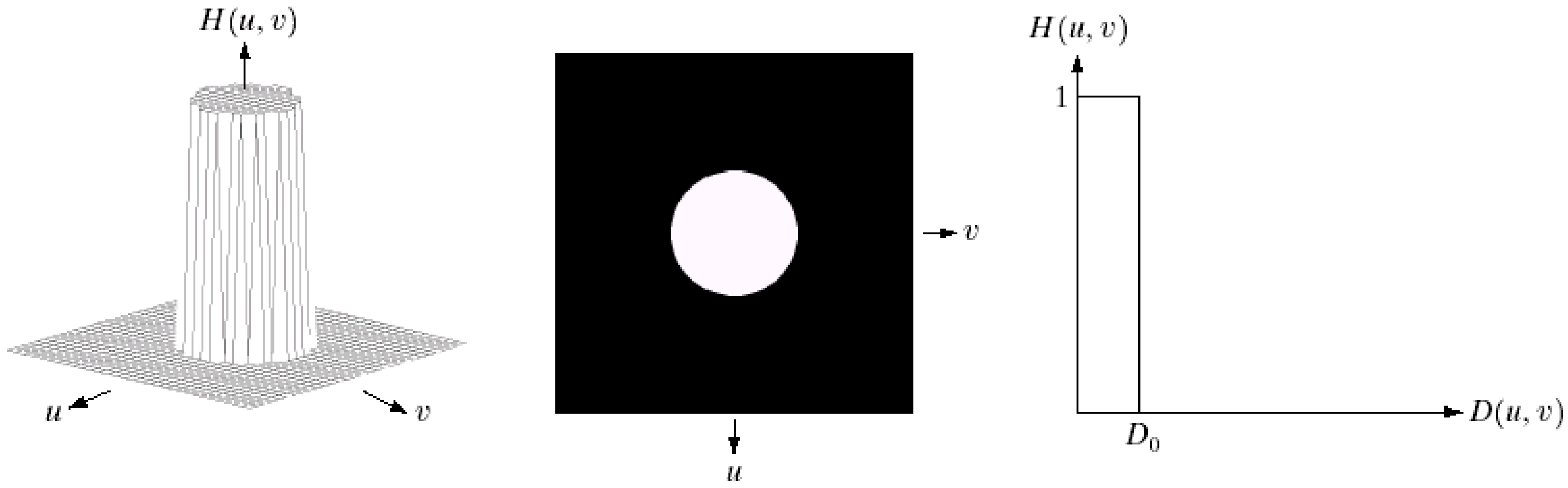
$$G(u,v) = H(u,v)F(u,v)$$

where $F(u,v)$ is the Fourier transform of the image being filtered and $H(u,v)$ is the filter transform function

Low pass filters – only pass the low frequencies, drop the high ones

Ideal Low Pass Filter

Simply cut off all high frequency components that are a specified distance D_0 from the origin of the transform



changing the distance changes the behaviour of the filter

Ideal Low Pass Filter (cont…)

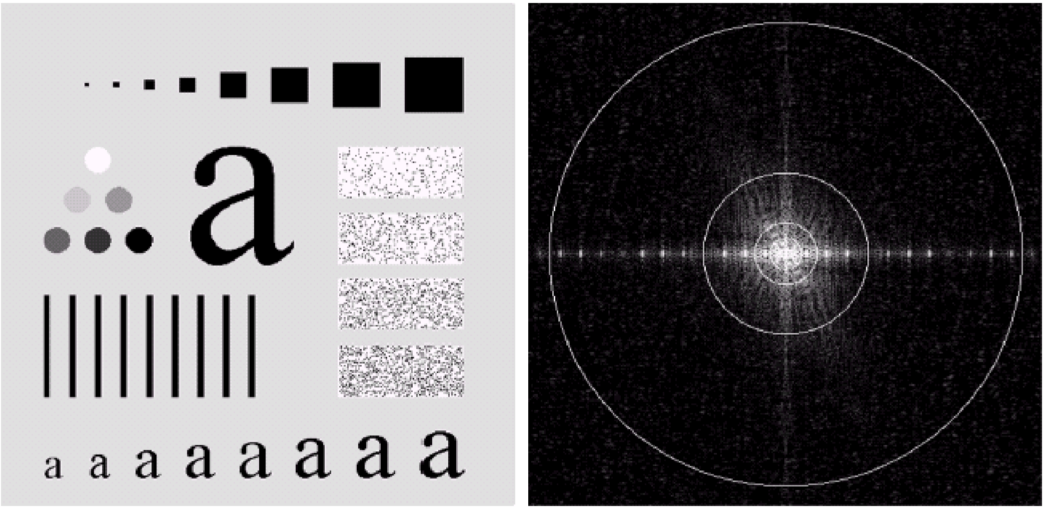
The transfer function for the ideal low pass filter can be given as:

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$$

where $D(u,v)$ is given as:

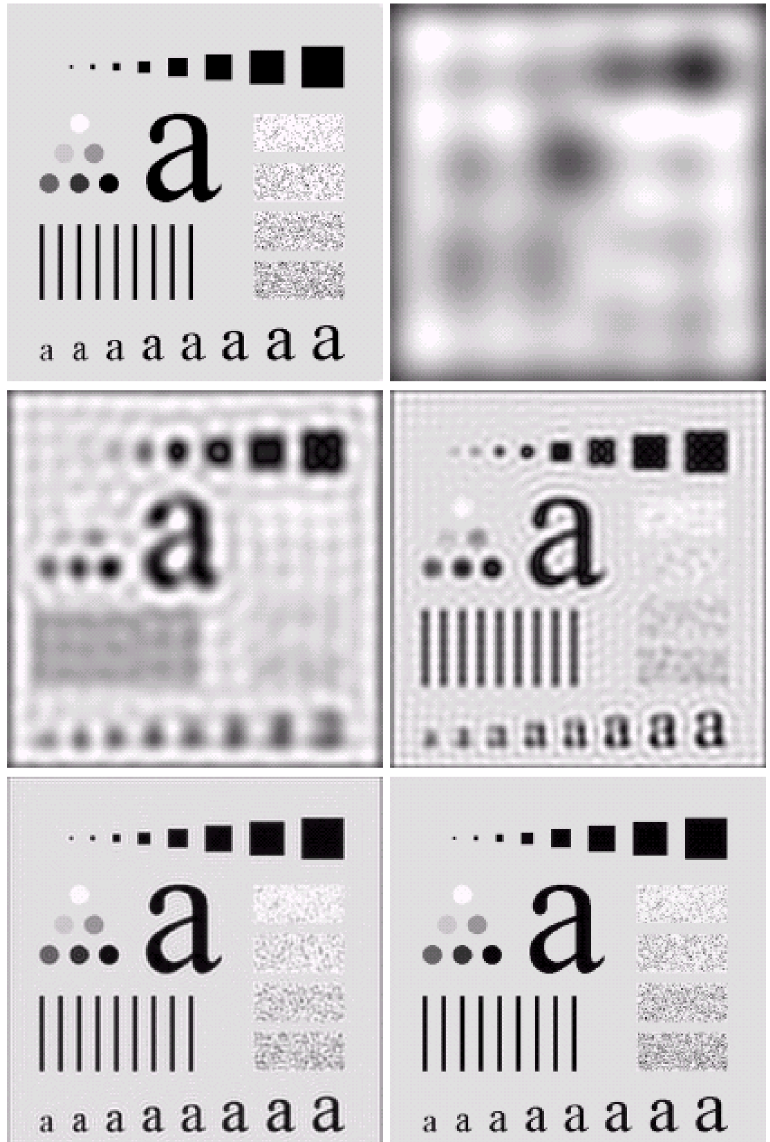
$$D(u,v) = [(u - M/2)^2 + (v - N/2)^2]^{1/2}$$

Ideal Low Pass Filter (cont…)



Above we show an image, it's Fourier spectrum and a series of ideal low pass filters of radius 5, 15, 30, 80 and 230 superimposed on top of it

Ideal Low Pass Filter (cont…)



Original image

Result of filtering with ideal low pass filter of radius 5

Result of filtering with ideal low pass filter of radius 15

Result of filtering with ideal low pass filter of radius 30

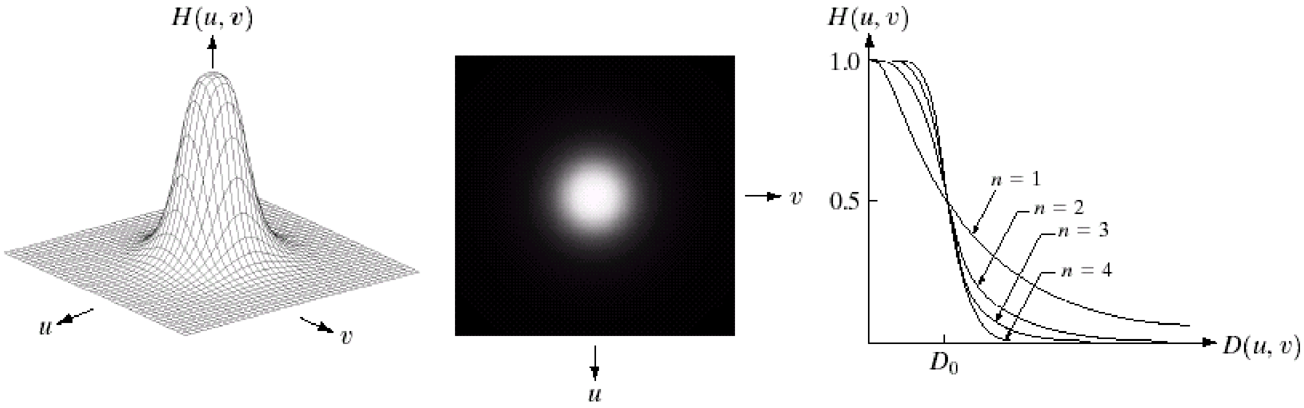
Result of filtering with ideal low pass filter of radius 80

Result of filtering with ideal low pass filter of radius 230

Butterworth Lowpass Filters

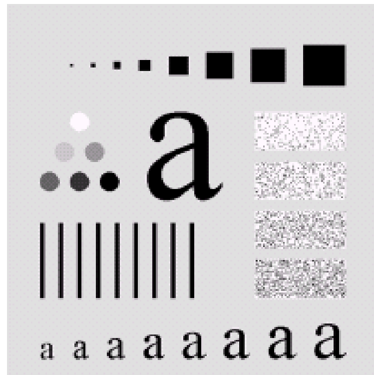
The transfer function of a Butterworth lowpass filter of order n with cutoff frequency at distance D_0 from the origin is defined as:

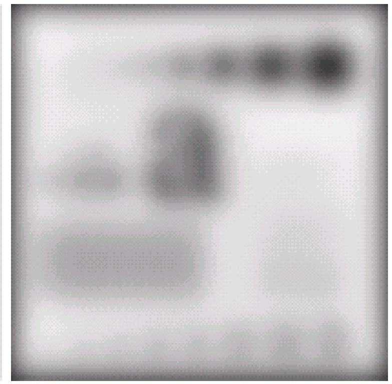
$$H(u,v) = \frac{1}{1+[D(u,v)/D_0]^{2n}}$$



Butterworth Lowpass Filter (cont…)

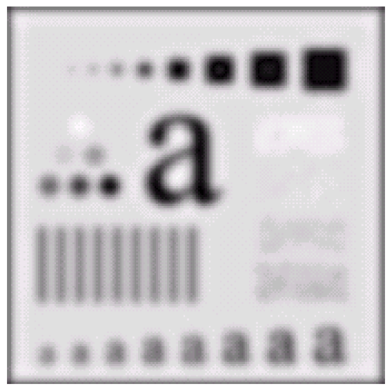
Original image

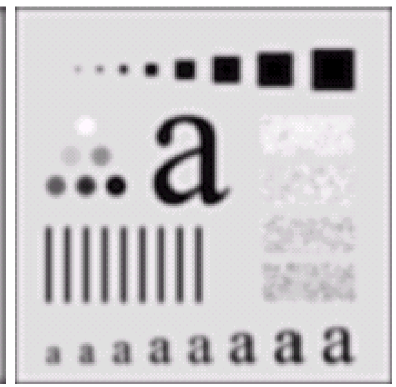




Result of filtering with Butterworth filter of order 2 and cutoff radius 5

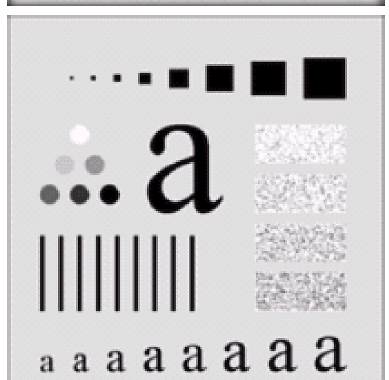
Result of filtering with Butterworth filter of order 2 and cutoff radius 15

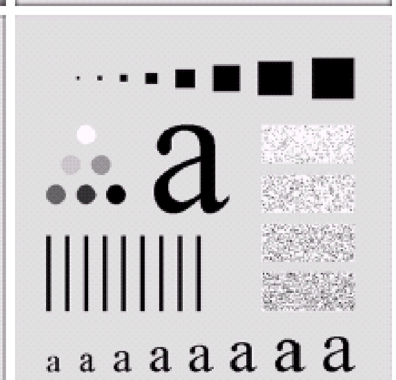




Result of filtering with Butterworth filter of order 2 and cutoff radius 30

Result of filtering with Butterworth filter of order 2 and cutoff radius 80



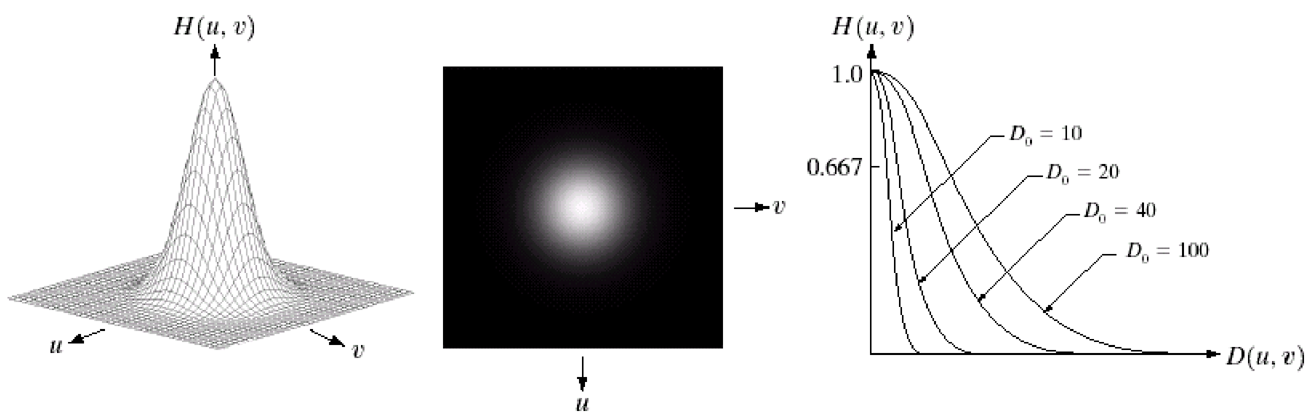


Result of filtering with Butterworth filter of order 2 and cutoff radius 230

Gaussian Lowpass Filters

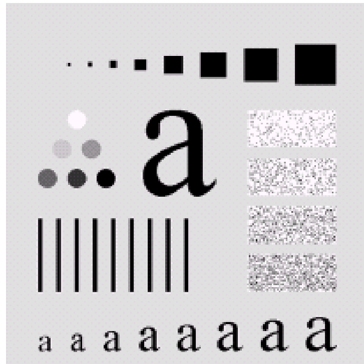
The transfer function of a Gaussian lowpass filter is defined as:

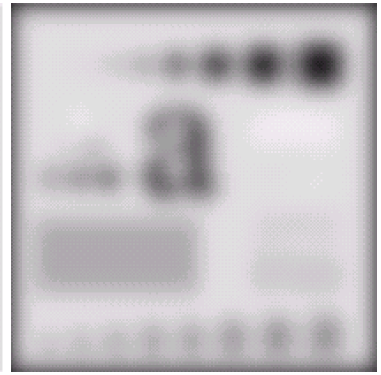
$$H(u,v) = e^{-D^2(u,v)/2D_0^{\ 2}}$$



Gaussian Lowpass Filters (cont…)

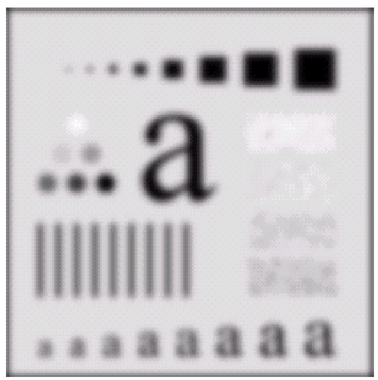
Original image

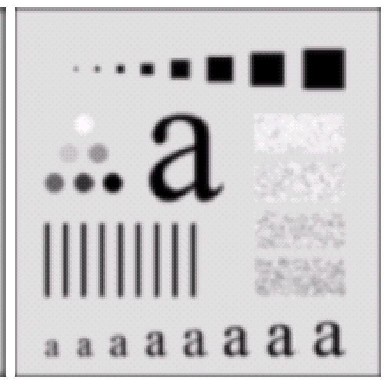




Result of filtering with Gaussian filter with cutoff radius 5

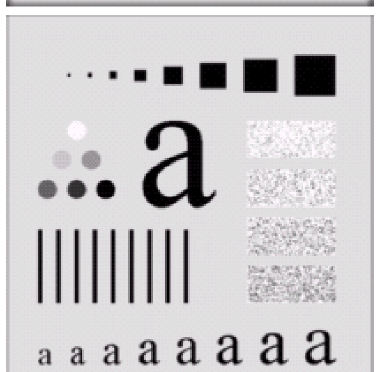
Result of filtering with Gaussian filter with cutoff radius 15

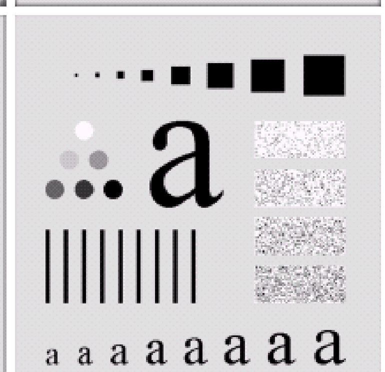




Result of filtering with Gaussian filter with cutoff radius 30

Result of filtering with Gaussian filter with cutoff radius 85

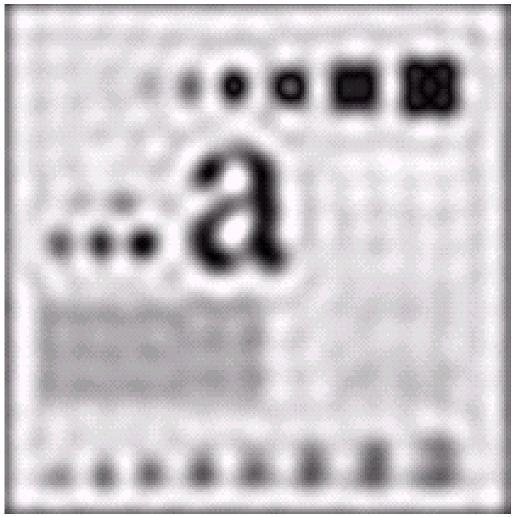


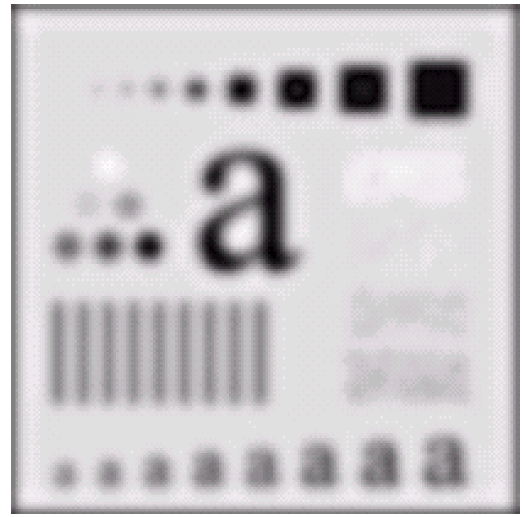


Result of filtering with Gaussian filter with cutoff radius 230

Lowpass Filters Compared

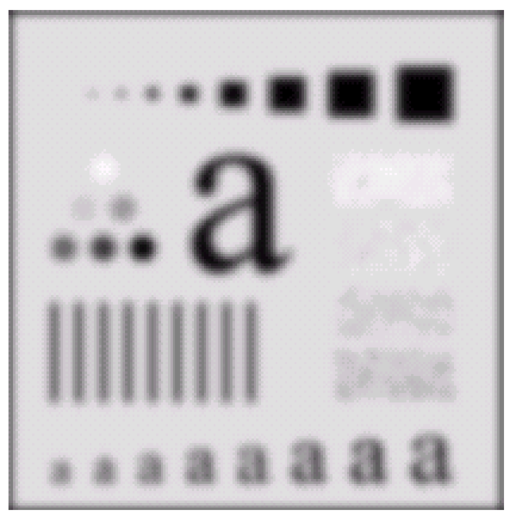
Result of filtering with ideal low pass filter of radius 15





Result of filtering with Butterworth filter of order 2 and cutoff radius 15

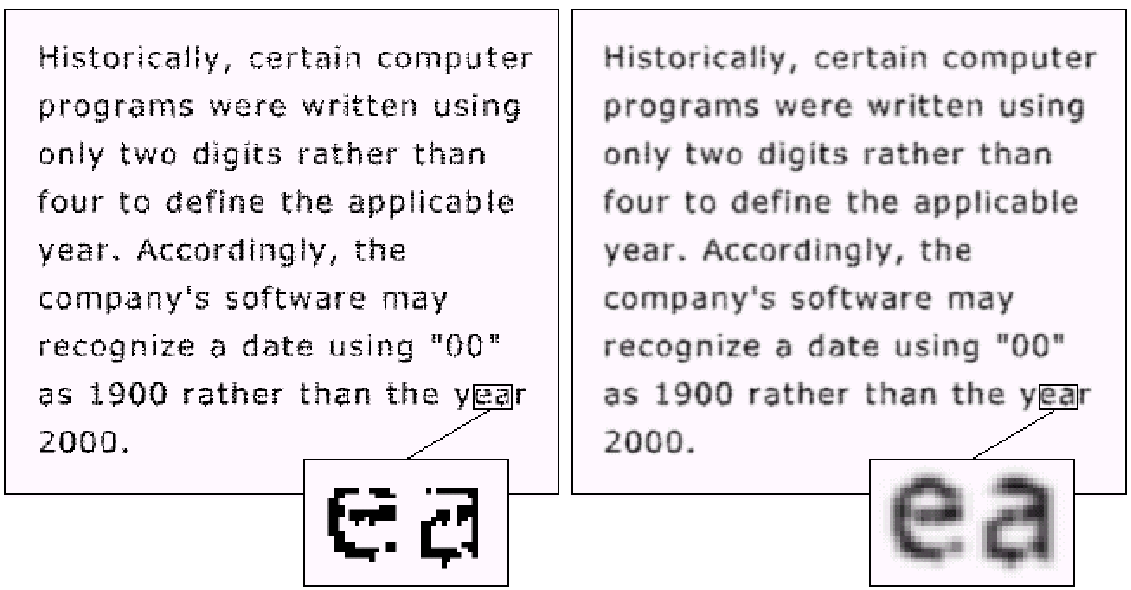
Result of filtering with Gaussian filter with cutoff radius 15



Lowpass Filtering Examples
Poor resolution

For the broken char. The human visual system can fill these gaps, but the M/C is not

A low pass Gaussian filter is used to connect broken text(blurring)

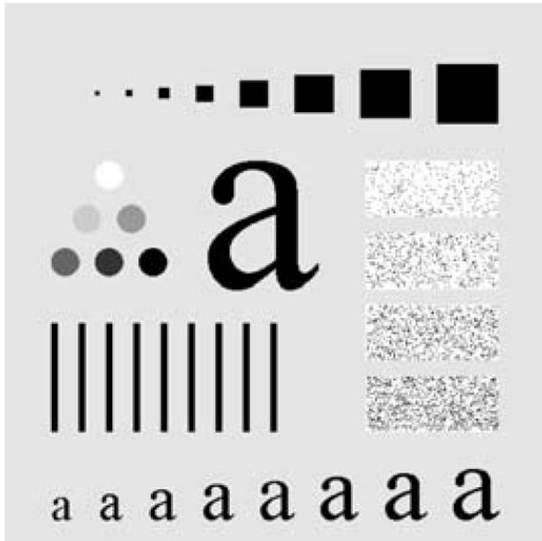


Lowpass Filtering Examples cosmetic Application

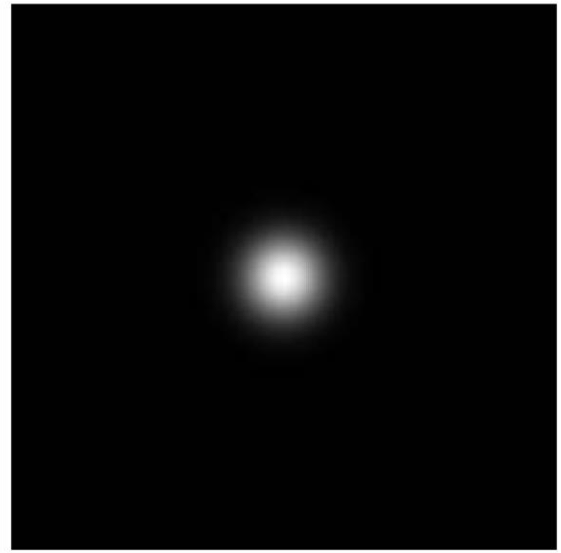
Different lowpass Gaussian filters used to remove blemishes in a photograph

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

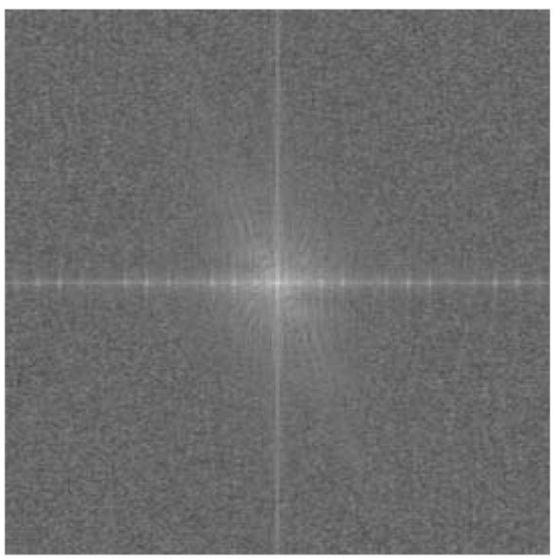
Lowpass Filtering Examples (cont…)



Original image



Gaussian lowpass filter



Spectrum of original image

Processed image

Sharpening in the Frequency Domain

Edges and fine detail in images are associated with high frequency components

High pass filters – only pass the high frequencies, drop the low ones

High pass frequencies are precisely the reverse of low pass filters, so:

$$H_{HP}(u, v) = 1 - H_{Lp}(u, v)$$

Sharpening Frequency Domain Filter:

Ideal highpass filter

$$H(u,v) = \begin{cases} 0 \text{ if } D(u,v) \le D_0 \\ 1 \text{ if } D(u,v) > D_0 \end{cases}$$

Butterworth highpass filter

$$H(u,v) = \frac{1}{1 + [D_0 / D(u,v)]^{2n}}$$

Gaussian highpass filter

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

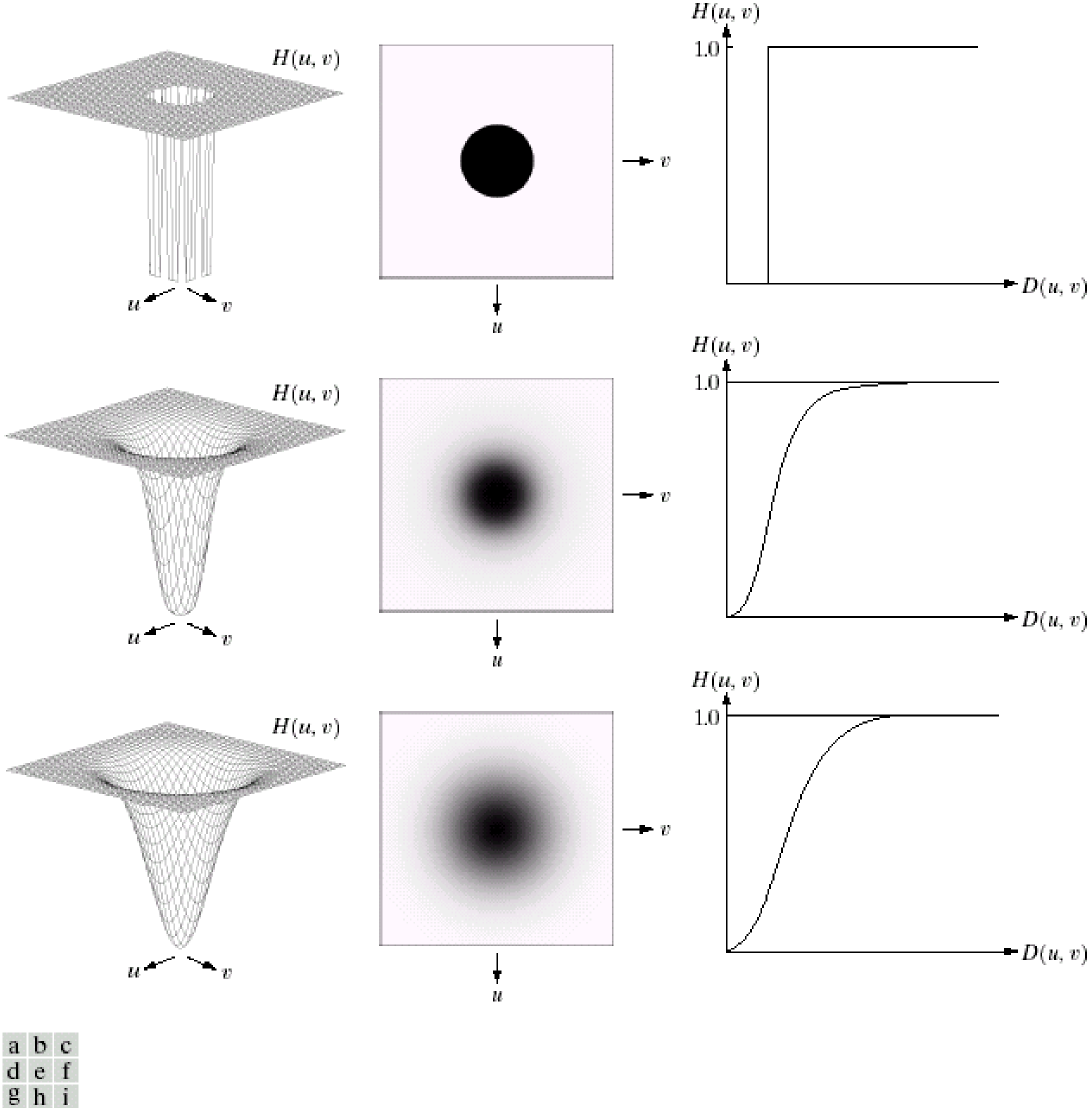


FIGURE 4.22 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

Spatial representation of Ideal, Butterworth and Gaussian highpass filters

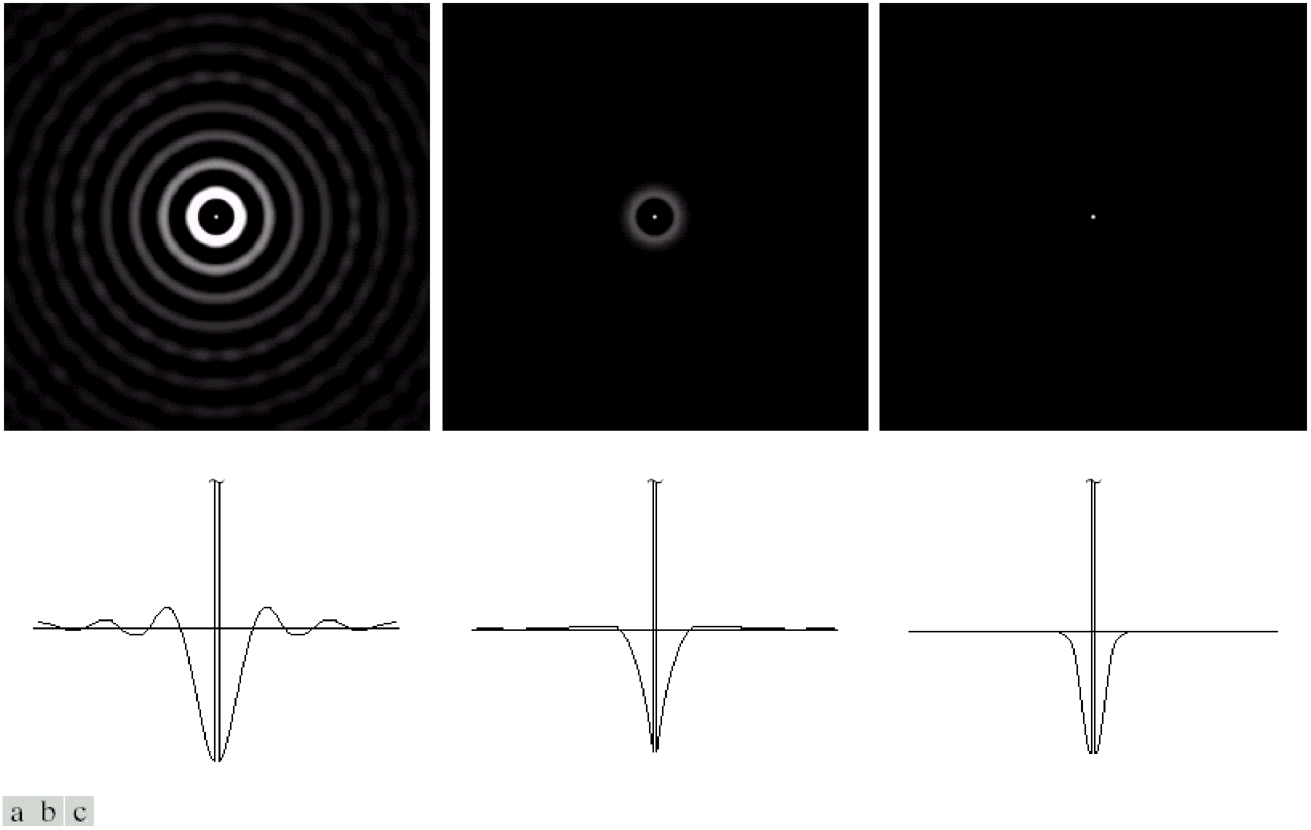


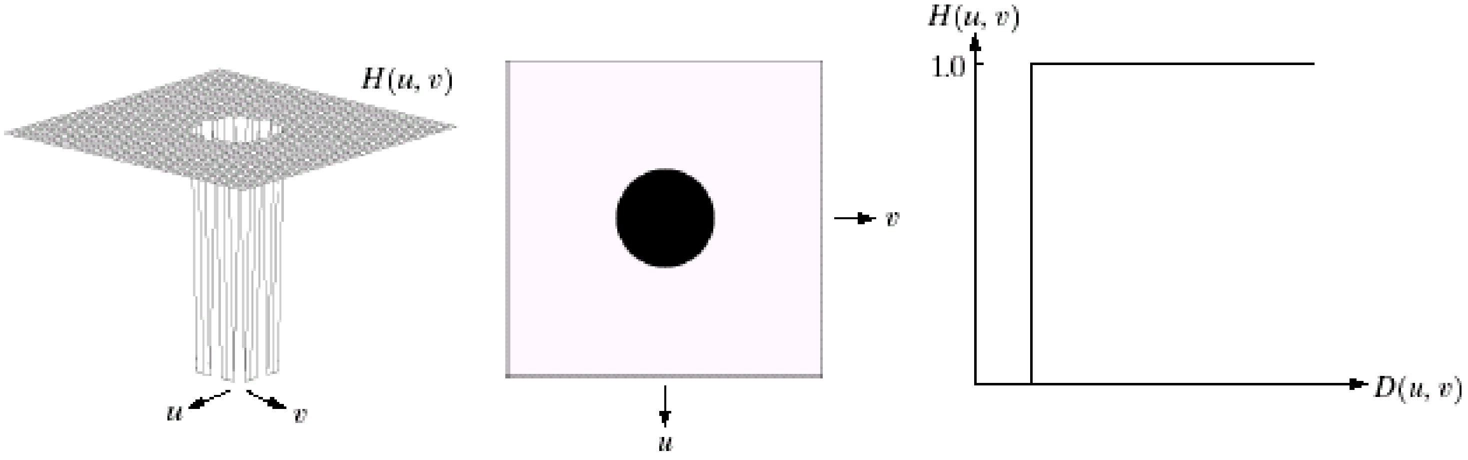
FIGURE 4.23 Spatial representations of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding gray-level profiles.

Ideal High Pass Filters

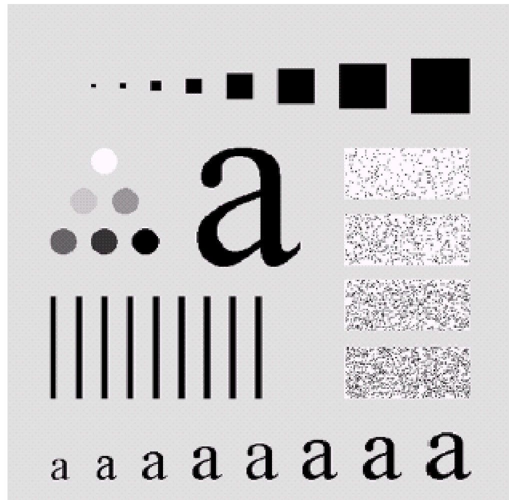
The ideal high pass filter is given as:

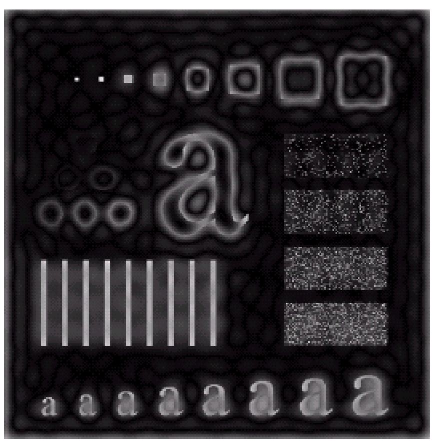
$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \leq D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

where D_0 is the cut off distance as before

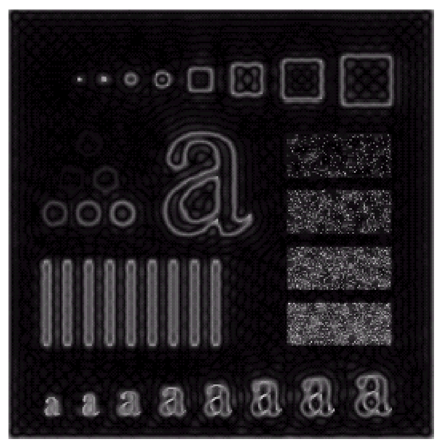


Ideal High Pass Filters (cont…)

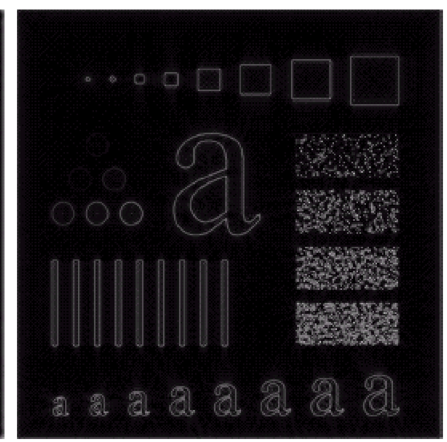




Results of ideal high pass filtering with $D_0 = 15$



Results of ideal high pass filtering with $D_0 = 30$



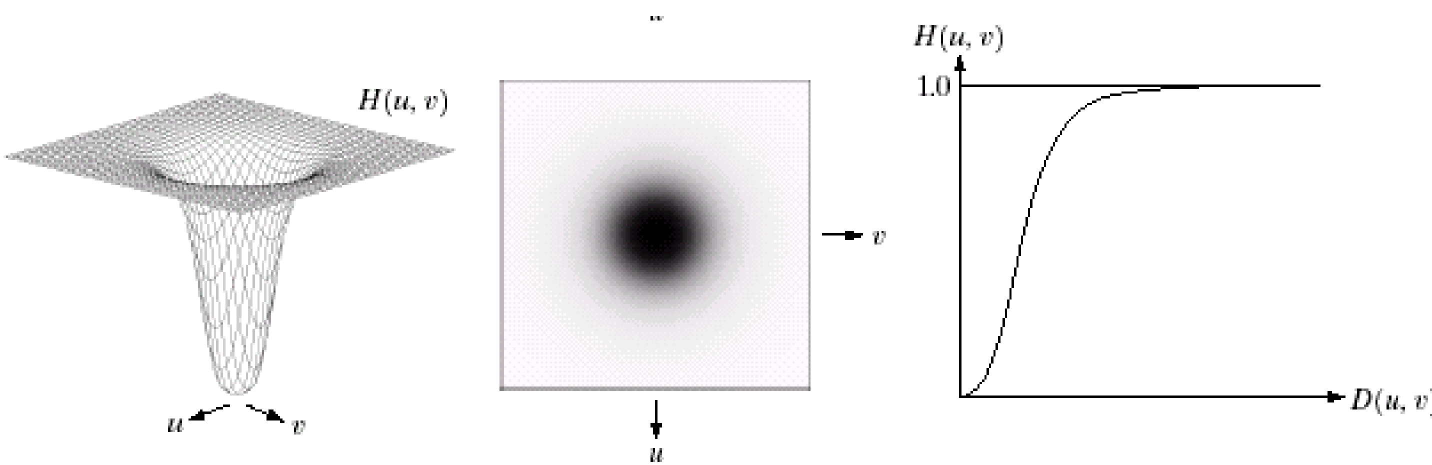
Results of ideal high pass filtering with $D_0 = 80$

Butterworth High Pass Filters

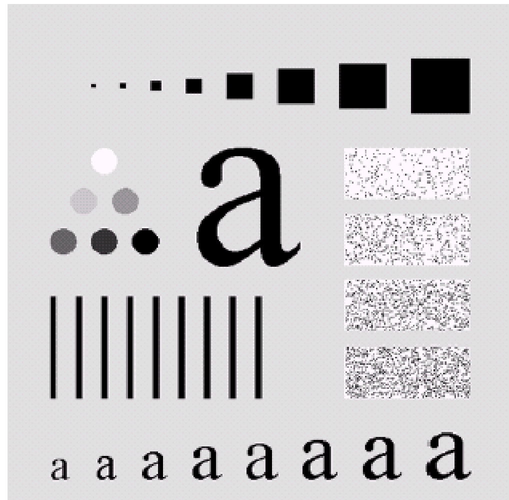
The Butterworth high pass filter is given as:

$$H(u,v) = \frac{1}{1+[D_0 / D(u,v)]^{2n}}$$

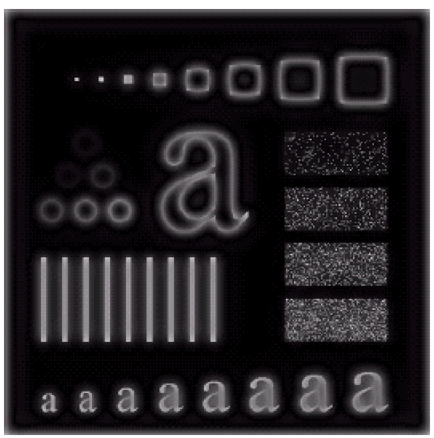
where n is the order and D_0 is the cut off distance as before

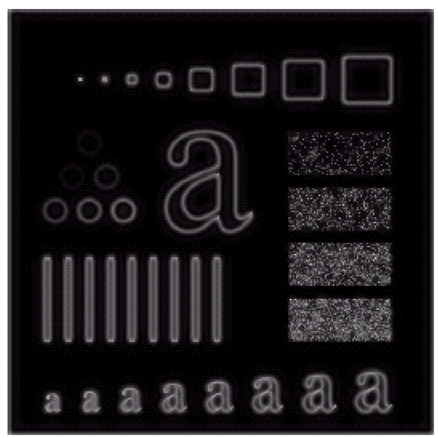


Butterworth High Pass Filters (cont…)

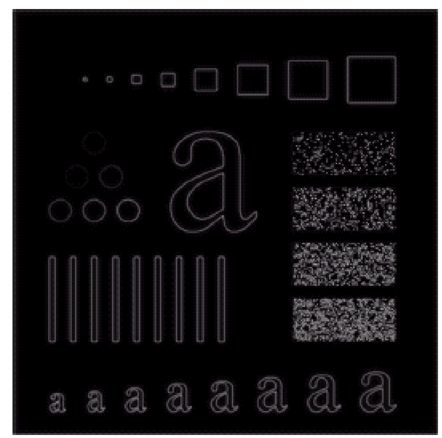


Results of Butterworth high pass filtering of order 2 with $D_0 = 15$





Results of Butterworth high pass filtering of order 2 with $D_0 = 30$



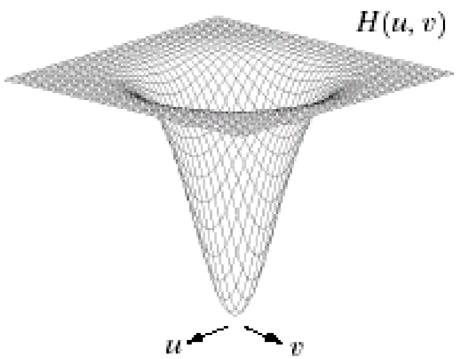
Results of Butterworth high pass filtering of order 2 with $D_0 = 80$

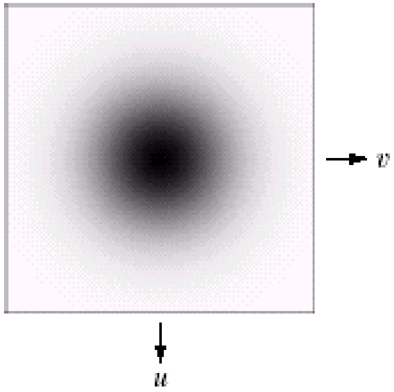
Gaussian High Pass Filters

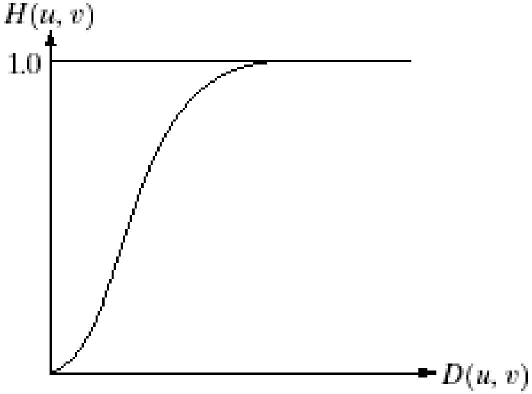
The Gaussian high pass filter is given as:

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

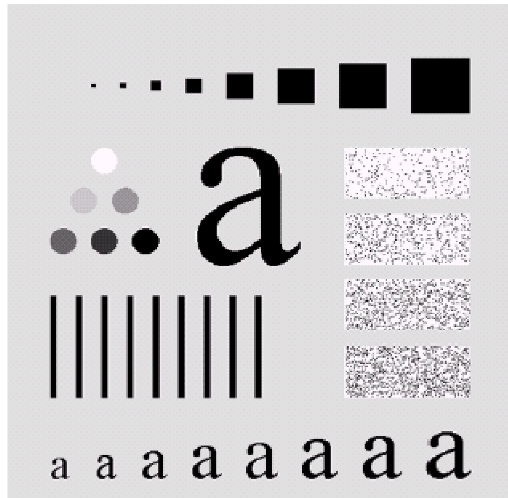
where D_0 is the cut off distance as before



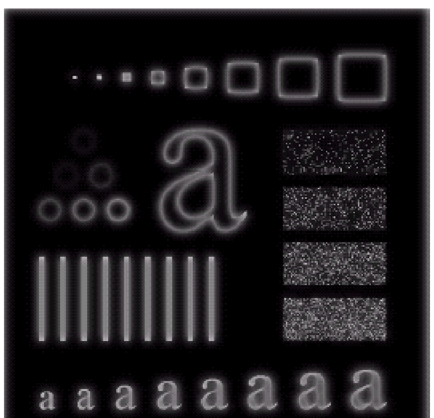


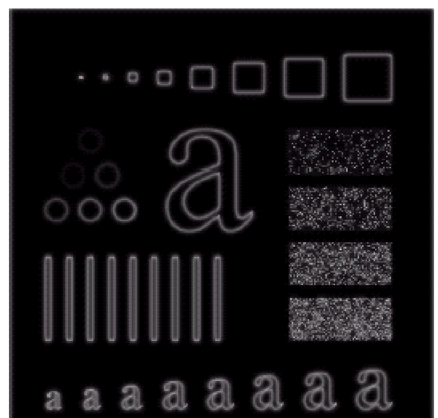


Gaussian High Pass Filters (cont…)



Results of Gaussian high pass filtering with $D_0 = 15$



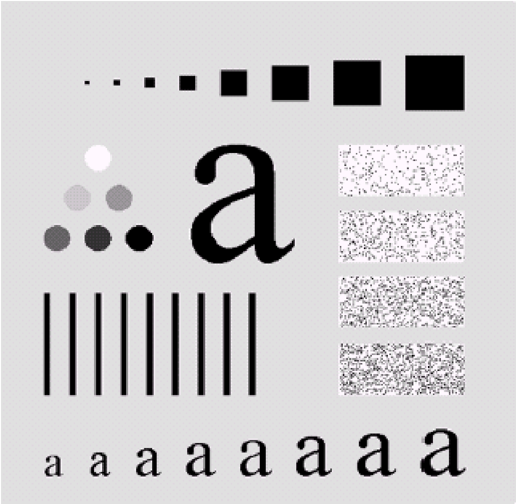


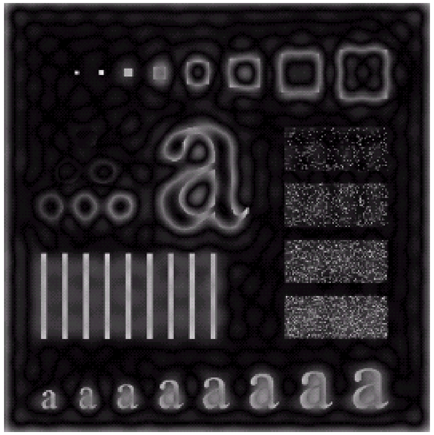
Results of Gaussian high pass filtering with $D_0 = 30$

Results of Gaussian high pass filtering with $D_0 = 80$

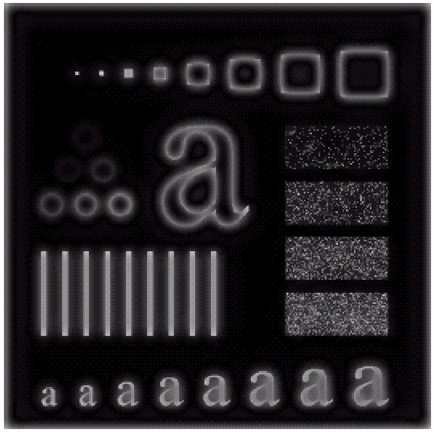
Highpass Filter Comparison

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

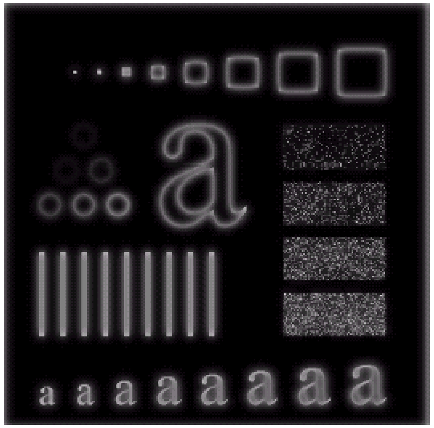




Results of ideal high pass filtering with $D_0 = 15$



Results of Butterworth high pass filtering of order 2 with $D_0 = 15$



Results of Gaussian high pass filtering with $D_0 = 15$